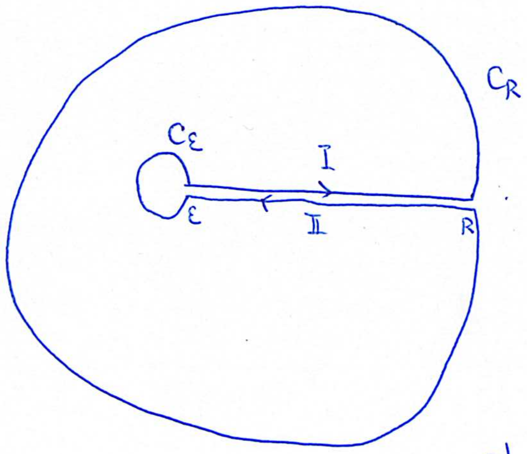
Lecture 22 and 23.

we also introduced the path contour to solve



$\int_0^{+\infty} \frac{x^{-a}}{x+1}\,dx$. branch cut in this example is $\arg \in [0, 2\pi)$. path I is on real axis, but is touched right above the x-axis, so argument on path I are all zero. path II is touched right below the x-axis, so arguments on II are all 2π.

we also studied type integral. That is

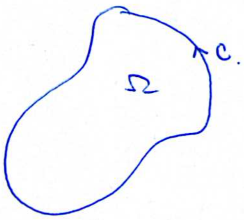
$$\int_0^{2\pi} F(\cos\theta, \sin\theta)\, d\theta$$

through change of variable $z = e^{i\theta}$. it can be reduced to

$$\int_{\{|z|=1\}} F\left(\frac{z+\frac{1}{z}}{2}, \frac{z-\frac{1}{z}}{2i}\right) \cdot \frac{1}{iz}\, dz.$$

this integral can be evaluated by Cauchy Residue.

App 3: argument principle.



f is meromorphic in Ω. $f \neq 0$ on C.

then $\frac{1}{2\pi}$ (total change of arguments along image of C under f)

$$= N - P$$

N = total # of zeros of f in Ω, counting multiplicities.

P = total # of poles of f in Ω. counting multiplicities

ex: $f(z) = z^2 + \frac{2}{z}$

Calculate $\frac{1}{2\pi i} \int_{|z|=1} \frac{f'}{f}$, $\frac{1}{2\pi i} \int_{|z|=4} \cancel{z^2} \frac{f'}{f}$.